The Standard Model and its symmetries
How theory and experiment meet.

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Abstract

In this thesis I will examine the Standard Model and her consequences. The complete theory which is the Standard Model will be developed and all its symmetries will be investigated. Symmetries, corresponding to conserved currents in nature, can occur both on a local and global scale. The local symmetries, mathematically denoted as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ will form the basis of the theory. As some symmetries get broken new physics will arise and in the end we will have a complete overview of this theory and its manifestation in High Energy Physics.

Contents

1. DUTCH SUMMARY

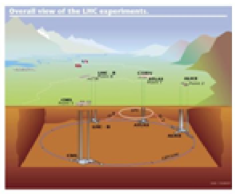
1.1. De symmetrieën van het Standaard Model

In dit project heb ik het formalisme van de theoretische hoge energie fysica HEF bestudeerd. De hoge energie fysica (HEF) heb ik de afgelopen vier jaar eigenlijk alleen leren kennen als het plaatje rechtsonder, waarin je het grootste experiment ter wereld ziet, gelegen in Geneve. Hier worden deeltjes versneld tot zeer hoge energie om daarna op elkaar te botsen. Met de energie die daarbij vrijkomt hopen natuurkundigen nieuwe, nu nog onbekende deeltjes en reacties te zien. Zoals alle natuurkunde wordt ook de HEF beschreven door een theorie met wiskundige notatie. De afgelopen jaren heb ik ook daar veel aan gedaan, zo weet ik nu dat de formules linksonder alles met elektriciteit en magnetisme beschrijven. Maar nooit ben ik toegekomen aan de theorie die beschrijft wat er in Geneve gebeurt. Het instituut in Nederland dat meewerkt aan dat grote experiment is NIKHEF, en daar op de theorie afdeling heb ik mijn project gedaan. Mijn begeleider heeft me laten zien hoe de HEF theoretisch beschreven kan worden door een enkele formule. Hoe deze formule afgeleid kan worden door middel van al dan niet "gebroken symmetrieën" heb ik bestudeerd en daarbij zal ik nu een kleine inleiding geven.

Natuurkunde valt het makkelijkst te beschrijven in de vorm van een Lagrangiaan, dat is een formule die gelijk is aan het verschil tussen kinetische en potentiële energie, $L = T - V$. Vanuit die Lagrangiaan kunnen alle bewegingsvergelijkingen van deeltjes worden afgeleid. Aan die formule kunnen we een paar dingen zien, onder andere dat de bewegingsvergelijkingen en de Lagraniaan niet veranderen als we ze vermenigvuldigen met een complexe e-macht en ook dat we er van alles bij op kunnen tellen. Beide dingen mogen we alleen doen als ze geen effect hebben op de natuurkunde die we proberen te beschrijven. We mogen dus alleen op die manier iets aan de Lagrangiaan veranderen zodat het iets verklaart in de natuur dat we kunnen waarnemen en niet al verklaart was. Het eerste, dat wat de Lagrangiaan onveranderd laat, noemen we een symmetrie van het systeem. De bekendste vrouw uit de theoretische natuurkunde, mevrouw Emmy Noether, heeft laten zien dat zo′n symmetrie overeenkomt met een behouden grootheid in de natuur.

Om het hele proces te beschrijven kost erg veel bladzijdes met formules maar uiteindelijk zal die simpele e-macht bijdragen aan het behoud van lading zoals we dat kennen in de natuur. Uit de uiteindelijke Lagrangiaan, die van "het Standaard Model", valt alles af te leiden wat met deeltjes te maken heeft. Daarmee heb ik nu het hele overzicht gekregen tussen theorie en experiment en snap op welke manier ze aan elkaar kunnen bijdragen!

$$\begin{aligned} \oint \mathbf{E}\cdot d\mathbf{A} &= \frac{q_{enc}}{\varepsilon_0} \\ \oint \mathbf{B}\cdot d\mathbf{A} &= 0 \\ \oint \mathbf{E}\cdot d\mathbf{s} &= -\frac{d\Phi_B}{dt} \\ \oint \mathbf{B}\cdot d\mathbf{s} &= \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc} \end{aligned} \quad \sim$$



2. INTRODUCTION

To start this paper I will first give a personal motivation for choosing this field of physics for this thesis. Then I will put this into a more physical formulation.

2.1. Personal Introduction

The idea for this thesis started in a reflection of my last three years as a bachelor student at the University of Amsterdam. I had enjoyed myself very much in all my classes but wondered how good a physicist I had become. For three years we saw pictures of CERN in our classes. We never went as far into it as I would have liked, though, to grasp the fundamental theoretical basics of what was going on there. We were told beautiful stories of that immense place, which I had visited when I was just 17 years of age. They even showed us a picture once of a single formula which could explain everything that was going on down there a few hundred feet below Geneva. But we did not get to examine the formula and were told we had to wait until we were ready. In our theoretical subjects we would get classical Lagrange theory and the basics of Electrodynamics. But they were never combined into a theory which connected to our more phenomenological particle physics classes. So after three years of study we seemed, to some extent, to be graduating without really knowing what we were doing.
That is why I decided to do my bachelor thesis on the theoretical particle physics. With the most appealing experiments at CERN, the world of particle physics is one of the fields of physics that is more popular with the general public. It is therefore that I did not want to graduate without knowing the theoretical physics that describes the elementary particles and their kinematics. To see whether there is equal beauty in the formulation of the theory as in the magnitude of the experiments!

2.2. Physical Introduction

I have been writing about my classes over the past years and experiments at CERN and how I want to match them in this project. But in a more physical formulation, what I will be trying to achieve is this: To grasp the theoretical formulation that describes High Energy Physics (HEP) and link that to experimental facts that we know from nature. An important principle that is used in this respect is symmetry. We speak of a symmetry when a symmetry-operation leaves a system that we are observing intact. Symmetry will be both the link between experimental results and the theory as the constraint on our theoretical formulation of nature.

2.3. General Setup

I will take you step by step into the world of theoretical HEP. First I will give an introduction to Lagrangian formulation and the part symmetries can play with respect to that. Then, when we have picked up enough tools, we can start with the construction of the Local Gauge Theory of the Standard Model. This will result in the single formula for everything in HEP. After this, we'll have seen a lot of symmetries and corresponding physics.

When the Standard Model is complete we will look a bit further and notice some other global symmetries and highlight some interesting details and consequences of what we have seen. At the very end I will direct a few words to the meaning of what we have done and the confidence we can have in this particular theory. Since this confidence will turn out to be pretty high we can then look briefly at extensions of this theory, instead of conflicting theories. But given the fact that we have no experimental knowledge on the scale beyond that of the Standard Model there will be very little to discuss. I will conclude my thesis with a final word of reflection on the Standard Model and the role it plays in improving my understanding of HEP.

3. THEORY

We will now get to the big physics part. Before we can actually start talking about symmetries and their consequences, I will first introduce a general outline of theoretical physics. I will introduce the tools we need to create the Standard Model and then we will finally get to its derivation. We will see which symmetries it contains and how they reflect on nature as we observe it in HEP.

3.1. Formalism

In HEP we usually look at collision and decay processes in which elementary particles interact. To describe these processes we use equations of motion that relate the energy, movement and mass of the different particles. In classical physics we can derive all these equations of motion from one single formula which we call the Lagrangian. This formula is given by the difference of the kinetic and potential energy:

$$L = T - V \tag{1}$$

We are always allowed to add extra potential terms that fit the system we are describing. In our case, where we will try to build the Lagrangian of a quantum field theory vacuum. We will use this to add terms that describe the vacuum as a scalar potential. What follows will be the interactions between these scalar particles and the other elementary particles. We derive the equations of motion by means of Hamilton's principle of least action. $S \equiv \int_{t_1}^{t_2} dt L(q, \dot{q})$ in which q is any variable of the Lagrangian and $\dot{q}$ its time derivative. Hamilton's principle states that $\delta S = 0$ which leads to the Euler-Lagrange equation, and this gives us a equation of motion for any variable q:

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \tag{2}$$

This is a powerful way of formulating physics. Out of one formula we will be able to extract all the physical information we want. When the Lagrangian has been put into its final form we will be able to distinguish different terms, like mass terms and interaction terms. These will result in equations that describe the interactions and lay limits for masses when provided with the necessary amount of constants. Because a Lagrangian will always depend on some physical quantities. Constants like $\hbar$ and α_{EM} add units and give a relative weight to some of the terms in the Lagrangian.

3.1.1. *Relativistic invariance*

The physics we are interested in is not classical. We will first include the theory of special relativity in our Lagrangian formulation to make it relativistic invariant. This way we will have a theory which produces the same results in every inertial frame.
To make this step we need to step from the three-dimensional Lagrangian to a four-dimensional field theory. Our new Lagrangian will become a function of fields ϕ and of their four-gradient $\frac{\partial\phi(x)}{\partial x^\mu} \equiv \partial_\mu\phi(x)$. In analogy to classical physics the equations of motion for these fields will be given by the Euler-Lagrange equation:

$$\frac{\partial L}{\partial\phi(x)} = \partial_\mu \frac{\partial L}{\partial(\partial_\mu\phi(x))} \tag{3}$$

3.1.2. *Noether and Symmetries*

I will now show, as Ms. Emmy Noether has done many years ago, that a symmetry in a system can be related to a conserved current. This is of special interest to physicists because conserved currents lead to conserved charges who are observables in nature. So, if we explain all conserved currents in nature by symmetries in our model, we will have great confidence our theory is a correct one. Note that this also implies that if we find a symmetry in our model it must always be related to a conservation law in nature. And, because of this fact we must always add all terms that obey the symmetries of our system, we cannot leave out the terms we cannot explain.

In a classical system with only one variable it is easy to see that if the right hand side of the Euler-Lagrange equation is zero, there is a conserved quantity between the brackets on the left hand side:

$$\frac{d}{dt}[\frac{\partial L}{\partial\dot{q}}] = \frac{\partial L}{\partial q} \tag{4}$$

If $\frac{d}{dt}\frac{\partial L}{\partial\dot{q}} = 0$ then $\frac{\partial L}{\partial\dot{q}} = Conserved.$

This can be generalized for a system of N variables. If the total differential of L is zero, $dL = \sum_{i=1}^{n} \frac{\partial L}{\partial q_i} dq_i = 0$. This then leads to the conclusion that, for any combination of dq_i that leaves $dL = 0$, there is a conserved current on the left hand side of:

$$\frac{d}{dt}[\sum_{i=1}^{n}(\frac{\partial L}{\partial\dot{q}_i} dq_i)] = dL \tag{5}$$

In our relativistic invariant case the transformation of $x_\mu \to (x_\mu)' = x_\mu + a_\mu$, with a_μ a very small displacement of the coordinate x_μ, will lead to the general expression of a local conservation law:

$$\partial_\mu\Theta^{\mu\nu} = 0 \tag{6}$$

with

$$\Theta^{\mu\nu} \equiv \frac{\partial L}{\partial(\partial_\nu\phi)}\partial^\mu - g^{\mu\nu}L \tag{7}$$

This shows the conservation of the stress-energy-momentum tensor $\Theta^{\mu\nu}$, take for instance $\mu = 0$ and $\nu \neq 0$ and you will find the conservation of momentum density in field ϕ.

There are more symmetries then just transformations of coordinates. We can find symmetries that leave the quantum mechanical expectation values in a system unchanged. These symmetries are best described by matrices U who act on vectors of fields: $\phi = U\phi$. The most simple example in this case is the multiplication by a complex power of e like e^{ia}, here $\dim(U) = 1$. Because of quantum mechanics this will have no physical consequences. To make it more interesting we can go even further and assume that there can be a different a at every point in space-time. Then our global symmetry will become a local one. We will see that these local symmetries can form a basis of our understanding of HEP and help us capture it in one Lagrangian. When there is a derivative in the Lagrangian, a term equivalent to $e^{ia(x)}$ will give us extra, initially unwanted, terms. After differentiation we will see $e^{ia(x)}(\partial_\mu \phi(x) + i(\partial_\mu \phi(x))\phi(x))$. The symmetry seems lost. To compensate this unwanted extra second term we introduce a powerful mechanism known as a local or gauge transformation.[1] We replace the ordinary derivative by a new one which compensates for the unwanted term with a new gauge vector field:

$$\partial_\mu \to \partial_\mu - iA_\mu(x). \tag{8}$$

The compensation is a vector field A_μ, which transforms under the local transformation as

$$A'_\mu = A_\mu + \partial_\mu a(x) \tag{9}$$

This will undo the extra term the local symmetry created at the cost of an extra earlier unknown vector field. In the upcoming chapters we will use this mechanism to construct exactly as many fields as we know in nature and at the same time we will be able to create the symmetries we need to explain the known conservation laws. Before we can start with this we need to work out a few more details that are involved in building a quantum field theory like the Standard Model.

3.1.3. Broken Symmetries

We have now seen a first way in which we can adapt a Lagrangian to insert more physics. When we adjust something in our theory which violates a symmetry we say that symmetry is broken. A broken symmetry is no longer a symmetry of the system. But there are more ways that symmetries can help us in our quest to grasp nature. There are symmetries that pretend to be completely local or global but are violated or broken in a certain domain. If this is the case it is useful to be able to perform perturbation theory on the terms in the Lagrangian that break down this symmetry. This is a tool to restrict any symmetry breaking term to a special domain and later show that this term indeed breaks your symmetry in some rare interactions resulting from that perturbative term.

$$L_{complete} = L_{symmetric} + \epsilon L_{broken} \tag{10}$$

When we insert something like this ϵ we must be aware that we require extra input to our theory. This ϵ does not follow from our theory but has to be experimentally determined by measuring the interactions it is related to.

An even more complicated but essential ingredient in gauge theories is spontaneous symmetry breaking. This will be completely worked out in the chapter on Higgs breaking. I will just state here that a symmetry which leaves the Lagrangian invariant but is broken down by the ground state of that system is called a spontaneously broken symmetry.

3.2. The Local Symmetries

We will now start with the derivation of the Standard Model Lagrangian, constructed by local symmetries. But of course there must be some starting point, this is the Dirac equation[2] .

$$L_\psi = -\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi \tag{11}$$

A Lagrangian as we introduced it: a kinetic term with one derivative and a potential. Here the potential is just the mass-term. This is one of the basics of quantum theories and will need no further introduction. A good introduction into this material can be found in the book Quarks and Leptons by Halzen and Martin[3].

3.2.1. Lorentz Symmetry

This is a good moment to point out the implications of Lorentz invariance with respect to the Dirac equation. Because the Dirac equation is essentially not more than a linear and relativistic analogue of the Schrodinger equation from quantum mechanics.
One of the most fundamental theories of physics is that of special relativity. It is based on the idea that nature acts in the same way independent of in which inertial moving frame it is studied. To accomplice this Einstein postulated his theory of special relativity in which he announced a finite speed of light. This theory of special relativity is accompanied by a system of transformation rules to change coordinates of space-time from one system to another. This can be done by matrix multiplication with the Lorentz matrix:

$$x^\mu = \Lambda^\mu_\nu x^\nu \tag{12}$$

When accelerating in the first special direction v_x the matrix will look like:

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v_x/c$
This in a undefined basis with μ and ν running over the four space-time indices. Vector and tensor fields need a transformation for each of their indices. These fields can be made into Lorentz invariant quantities like $F_{\mu\nu}(x)F^{\mu\nu}(x)$ that, as we will see, return later as physical quantities in the Lagrangian.
The fields we describe are also Lorentz invariant and transform in an equivalent way. This is done by a combination of Dirac matrices γ_μ:

$$\Lambda_{\alpha\beta} = [exp(\frac{\omega^{\mu\nu}}{8}[\gamma_\mu, \gamma_\nu])]_{\alpha\beta} \tag{13}$$

This works as follows:

$$\psi_a^{'}(x^{'}) = \Lambda_{\alpha\beta}\psi_\beta(x) \tag{14}$$

This Lorentz symmetry we must obey from now on.

3.2.2. Building $SU(2)_L \otimes U(1)_Y$

Having said that, we can now start with the creation of the Standard Model Lagrangian. The Dirac equation, Eqs. (11), will be our starting point. We then will add terms and discover symmetries in them. These symmetries can be mathematically ordered in symmetry groups. In this paragraph we will see the construction of the first two groups $SU(2)_L \otimes U(1)_Y$ that will be responsible for the quantum electrodynamics and the weak nuclear force.

The group $SU(N)$ stands for Special Unitary group of dimension N, the collection of all N by N unitary matrices with determinant 1. The $U(N)$ group does not require that last restriction. In the Dirac equation, Eqs (11), we can require the local symmetry $\psi \rightarrow (\psi)^{'} = e^{iq\epsilon(x)}\psi$. We can compensate the effects a normal differentiation has on this symmetry by means of an adapted, so-called covariant derivative:

$$D_\mu\psi(x) = (\partial_\mu - iqA_\mu(x))\psi(x) \tag{15}$$

The transformation of A_μ as follows:

$$A_\mu \rightarrow A_\mu^{'} = A_\mu + \partial_\mu\epsilon \tag{16}$$

Combined this leads to the following symmetry and the introduction of the first vector field:

$$\begin{aligned} D_\mu\psi(x) \rightarrow (D_\mu\psi(x))^{'} &= (\partial_\mu\psi)^{'} - (iqA_\mu\psi)^{'} \\ &= e^{iq\epsilon(x)}(\partial_\mu\psi + iq\partial_\mu\epsilon\psi - iq(A_\mu\psi)^{'}) \\ &= e^{iq\epsilon(x)}(\partial_\mu\psi - iqA_\mu\psi) \end{aligned} \tag{17}$$

Now we have created this covariant derivative we can extend the Lagrangian.

$$L_\psi = -\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi + iqA_\mu\overline{\psi}\gamma^\mu\psi \tag{18}$$

With this vector field we can make another Lorentz invariant term that is also invariant under (16), which we should add before we have finished off the Lagrangian to this stage:

$$L_A = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\mu A_\nu)^2 = -\frac{1}{4}F_{\mu\nu}^2 \tag{19}$$

This $F_{\mu\nu}$ is the electromagnetic field strength tensor which we recognize again in the field equation that follows from $\partial L/\partial A_\mu$ and produces the electromagnetic four-vector current:

$$\partial^\nu F_{\mu\nu} = iq\overline{\psi}\gamma_\mu\psi \equiv J_\mu \tag{20}$$

The last result is actually the covariant notation of two of the Maxwell equations, see page 3.

Now we have seen the first easy steps of forming a Lagrangian. We will now turn to a more complicated phase transformation. This time the transformation will no longer depend on only one parameter but on more, this will make the transformation lose its abelian nature. As before we will write the matrices of these transformations as powers of e. To represent them in a correct way we write $e^{\xi^a t_a}$ where t_a are the generators of the group defined by the representation of ψ, and ξ^a are the set of linear independent parameters of the group.
Now we will, inspired by iso-spin invariance, create in the same way a Lagrangian for $SU(2)$ which is represented by just these transformations $U = e^{\xi^a t_a}$. We get this idea because we are aware of a global iso-spin invariance, namely that we are free to choose a basis for nucleons p and n. We would like to see if this transformation holds on a local level and see what consequences it will produce.
As before, the fact that we introduced symmetry dependent on x_μ will force us to create another covariant derivative which contains a new vector field W_μ:

$$D_\mu \psi \equiv \partial_\mu \psi - g W_\mu \psi \tag{21}$$

With,

$$W_\mu \equiv W_\mu^a t_a \tag{22}$$

hence there are as many W_μ^a as there are generators of the symmetry group. So far we have done nothing different from the one dimensional case, the parameter g is the charge we give to this vector field as we did before with the electromagnetic q, they are a measure for the strength of the phase transformation at hand. Because of the non-abelian character of this transformation the field strength tensor changes:

$$G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g[W_\mu, W_\nu] \tag{23}$$

This will result in some changes to the field equations but as well in the decomposition of $G_{\mu\nu}$. Namely, since it must transform covariantly like; $G_{\mu\nu} \rightarrow G'_{\mu\nu} = U G_{\mu\nu} U^{-1}$ and both W_μ and the commutator can be decomposed by t_a, so should $G_{\mu\nu}$:

$$G_{\mu\nu} = G^a_{\mu\nu} t_a \tag{24}$$

This will split the field W_μ into fields W_μ^a with a being the number of generators. In the case of $SU(2)$ this will result in 3 W-fields because the dimension of $SU(N)$ is always $N^2 - 1$. Inserting this in the Dirac equation will give us:

$$L = -\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi + g W_\mu^a \overline{\psi}\gamma^\mu t_a \psi \tag{25}$$

in which t_a are the three generators of $SU(2)$, equivalent to the Pauli matrices:

$$t_1 = \frac{1}{2}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, t_2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, t_3 = \frac{1}{2}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \tag{26}$$

At this point we have created 4 fields, W_μ^a and A_μ, as a result of $SU(2) \otimes U(1)$ symmetries. What we have not seen yet is any link to physical fields. To accomplish this we will need to make a side step. We cannot go on developing this theory without knowing where exactly we want to go.

3.2.3. Some facts from nature

Up to this point we took little notice on what actual fields we were working with. But now we are getting closer to the end of this section we must specify this clearly. In this domain of HEP we distinguish two types of elementary particles, they are either fermions or bosons. Fermions are half-integer spin elementary particles, they are the fundamental building stones of all matter. Bosons are full-integer spin particles, they can be represented by scalar fields.
Furthermore fermions are to be organized in two special ways, namely in so called left handed doublets and right handed singlets. We mean, that there is a fundamental difference between the same particles spinning left-handed in regard to their direction and the ones spinning in opposite direction. The difference is that the right handed fermions do not interact with the weak nuclear force while the left-handed do. This is something that should come forward from the theory we are developing and that is why we will split our Lagrangian into left-handed and right-handed terms:

$$L_{fermions} = -\sum_{i}^{n_{left}} \overline{\psi_L^i}\gamma_\mu \partial^\mu \psi_L^i - \sum_{i}^{n_{right}} \overline{\psi_R^i}\gamma_\mu \partial^\mu \psi_R^i \tag{27}$$

Here we distinguish between the number of left and right handed fermions because most theories build on the experimental fact that there are no right handed neutrinos. Measurements of neutrino oscillations imply that neutrinos do have a right-handed part, but they cannot couple to any gauge field. This however is a subject we will not fully examine at this stage. To split the fermions we define left and right handed projections out of the gamma matrices:

$$P_R = \frac{1}{2}(1+\gamma_5), P_L = \frac{1}{2}(1-\gamma_5) \tag{28}$$

with $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. Putting this into the starting Dirac equation and simplifying we get:

$$\begin{aligned} L &= -\overline{\psi}(P_R + P_L)(\partial + m)(P_R + P_L)\psi \\ &= -\overline{P_L\psi}(\partial)P_L\psi - \overline{P_R\psi}(\partial)P_R\psi - m\overline{P_R\psi}P_L - m\overline{P_L\psi}P_R \\ &= -\overline{\psi_L}\partial\psi_L - \overline{\psi_R}\partial\psi_R - m\overline{\psi_R}\psi_L - m\overline{\psi_L}\psi_R \end{aligned} \tag{29}$$

Here we see that the kinetic terms are separated for each handness, or "chirality", of fermion and the mass terms are mixed. Now that we have specified this part we will address a final word on charges. We know what charge is in electromagnetism and we saw it can be seen as the strength of the gauge phase transformation as well. This first charge, q, is unfortunately not the fundamental charge that we can straightforward produce from our theory. We observe that each doublet or singlet carries a so called hypercharge, Y, which is twice the average charge of the multiplet. It is related to normal charge by the Gell-Mann-Nishijima formula $Q = I_W + \frac{1}{2}Y$. This hypercharge is what we will be using as generator of the $U(1)$ group, as it depends on the fermion-wave-function at hand. As q disappears we will introduce another term together with Y which will represent the strength of the transition $(g^{'}/2)$, in which the half is just introduced for esthetical reasons. To underline this we will rename A_μ to B_μ. We can reformulate what we did in the last paragraph in these new terms, below you can see the rewritten covariant derivative:

$$D_\mu = \partial_\mu - igW_\mu^a t_a^{L,R} - i(g^{'}/2)B_\mu Y \tag{30}$$

Where t_a works differently on left and right handed fermions.

3.2.4. Higgs mechanism

We will now continue with something we have not seen before, as we will introduce a complex doublet of two scalar fields with hypercharge +1:

$$\phi \equiv \begin{pmatrix} \phi^1 \\ \phi^0 \end{pmatrix} \tag{31}$$

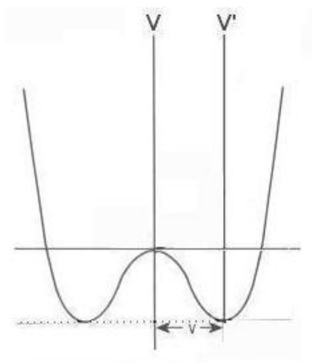
We assume this field interacts in a normal way and therefore we add a general potential, one with a single and quadratic term which we add to the Lagrangian that follows from equation (30) :

$$V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + \mid \lambda \mid (\phi^\dagger \phi)^2 \tag{32}$$

This is also where we add the interaction term between the new scalar fields and the left and right handed fermions. We assume they undergo Yukawa coupling, which is a highly symmetric Lorentz invariant way of coupling scalar fields to fermion fields.

$$L_{Yukawa} = \sum_{f=0}^{n_f} -G_f[\overline{\psi_R}(\phi^\dagger \psi_L) + (\overline{\psi_L}\phi)\psi_R] \tag{33}$$

Here G_f is the coupling constant for each family of fermions.
Then follows the real piece of art, developed by Prof. Peter Higgs. If we assume that $\mu^2 < 0$ the vacuum $< \phi >_0$ will no longer be at zero. This means the symmetries we have introduced so far get spontaneously broken. The new vacuum, or ground state as we may call it, will be at $< \phi >_0 = (0, v/\sqrt{2})$ with $v = \sqrt{-\mu^2 / \mid \lambda \mid}$. What actually happened is more clearly shown in the next figure:



Here we see the spontaneously broken potential $V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + \mid \lambda \mid (\phi^\dagger \phi)^2$, where the vacuum state jumps from 0 to $+v$.

We see that because of the jump from zero to v the symmetry of the system has disappeared. What we now want to do is to base our theory on the new replaced vacuum and see what this gives us as result. We need to find a generator that leaves the new ground state intact.

None of the four old generators t_a and Y succeed in this:

$$
\begin{aligned}
Y < \phi >_0 &= +1 < \phi >_0 \neq 0 \\
t_1 < \phi >_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
t_2 < \phi >_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
t_3 < \phi >_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\end{aligned} \tag{34}
$$

We notice however that the familiar linear combination from Gell-MannNishijima $(t_3 + Y)$ does! We have seen that a generator that leaves the vacuum intact generates a boson field. Now we see that the generator Q generates a field which we can relate to the familiar EM photon field.
The rewards this approach gives us will now come to light. When we observe what happens to the other generator we will see three other fields arise and three massive boson particles will be associated with those fields as I will now show.
We will have to expand the Lagrangian around the new vacuum because that will describe the physics correctly. This is done by rescaling the terms with a factor $e^{\frac{i\xi^a t_a}{2v}}$ with ξ^a and t_a respectively the generators and parameters and introducing η, a new fundamental scalar field boson resulting from the expansion:

$$
\begin{aligned}
L \rightarrow L^{'} &= e^{\frac{-i\xi^a t_a}{2v}} L \\
R &\rightarrow R \\
A_\mu &\rightarrow A_\mu^{'} \\
t_a \cdot W_\mu^a &\rightarrow t_a \cdot W_\mu^{a'} \\
\phi \rightarrow \phi^{'} &= e^{\frac{-i\xi^a t_a}{2v}} \phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}
\end{aligned} \tag{35}
$$

At this point we will put everything we did in the last paragraphs together. I will first give the Lagrangian terms containing the scalar fields as we saw them on the last page. Then, when we have cleared that up, we will perform a final redefinition of the bosons involved. We had:

$$
\begin{aligned}
L &= -\overline{\psi_L}\partial\psi_L - \overline{\psi_R}\partial\psi_R - m\overline{\psi_R}\psi_L - m\overline{\psi_R}\psi_R \\
D_\mu &= \partial_\mu - igW_\mu^a t_a^{L,R} - i(g^{'}/2)B_\mu Y
\end{aligned} \tag{36}
$$

this becomes when performing the shift:

$$
\begin{aligned}
L_{scalar} = &\frac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \mu^2\eta^2 \\
&+ \frac{v^2}{8}(g^2 \mid W_\mu^1 - iW_\mu^2 \mid^2 + (g^{'}B_\mu - gW_\mu^3)^2)
\end{aligned} \tag{37}
$$

What we can see here is that there is a kinetic and massive term for the η particle, which implies $M_\eta^2 = -2\mu^2$. In the lower line we see the W-fields and the B_μ.

We can rewrite this lower line by means of mixing W-fields one and two and the W-thee field with the B_μ-field to create a physically more meaningful equation. We introduce the physical fields $W^\pm$ and A_μ:

$$\begin{aligned} W_\mu^\pm &\equiv \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}} \\ Z_\mu &\equiv \frac{-g^{'}B_\mu + gW_\mu^3}{\sqrt{g^2+g^{'2}}} \\ A_\mu &\equiv \frac{-g_\mu^B + g^{'}W_\mu^3}{\sqrt{g^2+g^{'2}}} \end{aligned} \tag{38}$$

These linear combinations of the theoretical fields are defined this way because of the physical fields we observe. There are three massive ones and one massless. By adding and subtracting the definitions of the bosons given above, we find the last term in our Lagrangian which we can now write like this:

$$\begin{aligned} L_{scalar} = \cdots + \frac{v^2}{8}(g^2 \mid W_\mu^1 - iW_\mu^2 \mid^2 + (g^{'}A_\mu - gW_\mu^3)^2) &= \frac{g^2v^2}{8}(\mid W_\mu^+ \mid^2 + \mid W_\mu^- \mid^2) \\ &+ \frac{1}{8}\frac{g^2v^2}{\frac{g^2}{g^2+g^{'2}}} \mid Z^0 \mid^2 + L_{interactions} \end{aligned} \tag{39}$$

Because there are quite a few of these interaction terms between the four bosons and the fermions I did not include them right away. I will introduce them one by one: The interaction terms originate from the same starting point as the mass terms, elegantly written:

$$L_{interaction} = -\overline{\psi_R}(\partial_\mu - i(g^{'}/2)B_\mu Y)\psi_R - \overline{\psi_L}(\partial_\mu - igW_\mu^a t_a^{L,R} - i(g^{'}/2)B_\mu Y)\psi_L \tag{40}$$

Then filling in the new fields we get for the $W_\mu^\pm$-part:

$$L_{W^\pm} = \sum_{f=0}^{n_f} \frac{-g}{\sqrt{2}}(\overline{f_L^0}\gamma^\mu f_L^1 W_\mu^+ + \overline{f_L^1}\gamma^\mu f_L^0 W_\mu^-) \tag{41}$$

Here we sum over all the fermion families so we do not have to write down all three. When I speak of f^n I mean the n-th part of the fermion multiplet. Then we can in a similar way introduce the neutral boson interaction terms:

$$\begin{aligned} L_{A_\mu,Z_\mu} = \sum_{f=0}^{n_f}[&\frac{gg^{'}}{\sqrt{g^2+g^{'2}}}\overline{f^1}\gamma^\mu f^1 A_\mu - \frac{\sqrt{g^2+g^{'2}}}{2}\overline{f_L^0}\gamma^\mu f_L^0 Z_\mu \\ &+ \frac{Z_\mu}{\sqrt{g^2+g^{'2}}}(-g^{'2}\overline{f_R^1}\gamma^\mu f_R^1 + \frac{g^2+g^{'2}}{2}\overline{f_L^1}\gamma^\mu \overline{f_L^1})] \end{aligned} \tag{42}$$

What we have not done until now is inserting the transformation to the new ground state into the Yukawa coupling term[4]. This is as straightforward as possible because we know the transformation rules for left and right handed fermions. We get :

$$L_{Yukawa} = \sum_{f=0}^{n_f} -G_f\frac{(v+\eta)}{\sqrt{2}}[\overline{f_R}f_L + \overline{f_L}f_R] = \sum_{f=0}^{n_f} -\frac{G_f}{\sqrt{2}}(v+\eta)\overline{f}f \tag{43}$$

Which basically means that for every fermion f there is a mass $\frac{G_f v}{\sqrt{2}} = m_f$ and a coupling to the η field.
At this stage the Lagrangian that produces QED and the weak interaction is finished. All necessary terms have been developed and what is left for us to do is write it in the most elegant and comprehensible way.
From the first term in equation (42) we get A_μ as the photon field when we set $e = gg/\sqrt{g^2 + g'^2}$. To simplify the last neutral interaction terms we introduce the weak mixing angle θ_W:

$$g' = g \tan\theta_W \tag{44}$$

This implies that $\sqrt{g^2 + g'2} = g/\cos\theta_W$ and by this means it helps us to rewrite the neutral fields as a linear combination of t_3 and A_μ:

$$\begin{aligned} Z_\mu &= -B_\mu \sin\theta_W + t^3_\mu \cos\theta_W \\ A_\mu &= B_\mu \cos\theta_W + t^3_\mu \sin\theta_W \\ &or \\ B_\mu &= A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ t^3_\mu &= A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{aligned} \tag{45}$$

When inserting the mixing angle we find some more unexpected links:

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \tag{46}$$

$$\begin{aligned} g &= \frac{e}{\sin\theta_W} \\ g^{'} &= \frac{e}{\cos\theta_W} \end{aligned} \tag{47}$$

This predicts the small difference in the masses of the heavy bosons and links the strength of both local symmetries to the same parameter! Because of the central role this parameter plays in these local theories we usually write the entire Lagrangian in terms of θ_W.

Below I have written down what we have so far for the Standard Model Lagrangian. First we see the η terms. From now on we will call the η field the Higgs field, after its inventor. The potential for the Higgs field has spontaneously broken the $SU(2) \otimes U(1)$ symmetry but we have managed to rebuild it around the new ground state. This has left the local $U(1)$-gauge intact but has broken $SU(2)$. In the process of spontaneous breaking we have developed other terms in the Lagrangian. Terms that we were able to link to physical known fields; in the second row we see the coupling Yukawa terms, followed by the heavy boson mass terms. The last lines are filled with the fermion-boson interaction terms, written in terms of the weak mixing angle θ_W:

$$\begin{aligned} L_{SU(2)\otimes U(1)} &= \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &- \sum_{f=0}^{n_f} \frac{G_f}{\sqrt{2}}(v+\eta)\overline{f}f \\ &+ \frac{g^2 v^2}{8}(\mid W_\mu^+ \mid^2 + \mid W_\mu^- \mid^2) + \frac{1}{8}\frac{g^2 v^2}{\cos^2\theta_W} \mid Z^0 \mid^2 \\ &+ \frac{-g}{\sqrt{2}}(\overline{f_L^0}\gamma^\mu f_L^1 W_\mu^+ + \overline{f_L^1}\gamma^\mu f_L^0 W_\mu^-) \\ &+ f^1 \overline{f^1}\gamma^\mu f^1 A_\mu - \frac{g}{2\cos\theta_W}\overline{f_L^0}\gamma^\mu f_L^0 Z_\mu \\ &- \frac{g}{2\cos\theta_W}(2\sin^2\theta_W \overline{f_R^1}\gamma^\mu f_R^1 Z_\mu + (2\sin^2\theta_W - 1)\overline{f_L^1}\gamma^\mu \overline{f_L^1} Z_\mu) \end{aligned} \tag{48}$$

3.2.5. Where we stand

At this point it is wise to look back at where we stand. In this section of my project we developed the Lagrangian of the Standard Model, funded by local symmetries. We started with the Dirac equation, here we noticed two symmetries. Both symmetries generated scalar fields. By choosing the second symmetry to be of dimension two we got four fields, none of them were accompanied by a mass term in the Lagrangian. Those scalar fields were then mixed by the consequences of Higgs breaking into four scalar fields of which three had a massive term. This Higgs breaking was especially introduced for these purposes. The spontaneously breaking by this mechanism produced another field η, or the Higgs field. This consequence from Higgs′ theory must be present in nature, for else the theory is worthless. Unfortunately there has not yet been any measurement of a particle which measures the description. The reason why we still have major confidence in the Standard Model is that it predicts all data we know. It is therefore that nearly all physicists believe in this theory and assume that the Higgs particle will be found in the nearby future. The ATLAS and CMS experiments at CERN have been built precisely for this purpose.

Now let's see what else plays a role in HEP and is not yet included in our theory. What we are missing up to this point is the strong nuclear force. We have developed the weak and electromagnetic interaction but need to extend the theory to include the force that binds nucleons together.
Let us look at the fermions more specifically as well. Fermions manifest as stated above in doublets and singlets. We distinguish doublets of particles that interact strongly from those that do not. Those that do not we call leptons, there are three known lepton families. Those who do interact strongly we call quarks, of which we know three doublets as well. Other quantities we know about the fermions are shown more clearly in the following tabular:

	1	2	3
$+\frac{2}{3}$ $-\frac{1}{3}$	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$
-1 0	$\begin{pmatrix} e \\ v_e \end{pmatrix}$	$\begin{pmatrix} \mu \\ v_\mu \end{pmatrix}$	$\begin{pmatrix} \tau \\ v_\tau \end{pmatrix}$

(49)

The top line of doublets are the known quarks, the lower three the leptons. The first family is that of the up down quark doublet and the electron, practically all the matter around you is built from those particles because atoms are built from then. The particles in the second family are called the charm and strange quark and the muon. The third consists of the top and bottom quark and the tau particle. We organize them like this because charge is now aligned from top to bottom and mass from left to right. The first and lightest family is most common. The particles of the other families are heavier, and therefore less common. We need to develop a theory that describes six types of quarks and their interactions with one or multiple unspecified field(s). What we know about the strong interaction is that it binds the quarks together in a nucleon.
We must include another experimental fact as well, in weak decays a system decays three times more into a set of quark anti-quark then in that of a lepton anti-lepton. Physicists have introduced the concept of colour for this phenomena. They assume that there are three different colours of each quark and that a system decays equally into any type of fermion coming from the same family. Say the electron and the red, blue or yellow u-quark. It is not possible to measure colour in a direct way, we can never know what colour quark we are looking at. This is in fact the confined symmetry which we will use to extend the Standard Model and include the strong interaction in the next section.

3.2.6. The confined strong symmetry

The experimental evidence for the existence of colour is enormous. Not only in decay processes we find the number of quarks multiplied by a factor three but in cross sections and spin analysis of baryons as well. Let us assume colour is for quarks some type of charge and try to build a gauge theory based on it. We need to find a symmetry group that will produce the physics we want; three colours for every type of quark. We will take $SU(3)$ because it is the only group of dimension three that produces this result. As before we start with the Dirac equation because we want the physics we describe to be quantum mechanically and relativistically correct.

$$L = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2}trG^{\mu\nu}G_{\mu\nu} \tag{50}$$

The ψ stands for any of the quark colour-triplets. We already know the number of generators, or bosons this group will give us; $N^2 - 1 = 8$. This results in eight gauge fields b^n_μ in the covariant derivative:

$$D_\mu = \partial_\mu + \frac{ig}{2} b^n_\mu \lambda^n_\mu \tag{51}$$

The eight generators λ^n_μ can be represented by any three by three basis, like a normal set of isospin matrices of dimension three. We will not need an explicit representation here.
For the quadratic field term we must find a field strength tensor that is invariant under a transformation from $SU(3)$. This is a restriction we must take into account when we compute $G_{\mu\nu}$. We start with the basic form of a field tensor:

$$G_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\nu B_\mu - \partial_\mu B_\nu + ig[B_\nu, B_\mu] \tag{52}$$

Now we will check if this transforms in a sufficient way. To keep the Lagrangian invariant we need $G_{\mu\nu}$ to transform as $G_{\mu\nu} = UG_{\mu\nu}U^{-1}$ when $U \in SU(3)$. Filling in the transformation we get:

$$\begin{aligned} G'_{\mu\nu} &= U(\partial_\nu B_\mu - \partial_\mu B_\nu)U^{-1} \\ &+ ((\partial_\nu U)B_\mu - (\partial_\mu U)B_\nu)U^{-1} \\ &+ U(B_\mu(\partial_\nu U^{-1}) - B_\nu(\partial_\mu U^{-1})) \\ &+ \frac{i}{g}((\partial_\mu U)(\partial_\nu U^{-1}) - (\partial_\nu U)(\partial_\mu U^{-1})) \\ &+ igU[B_\nu, B_\mu]U^{-1} \\ &- U([U^{-1}(\partial_\nu U), B_\mu] - [U^{-1}(\partial_\mu U), B_\nu])U^{-1} \\ &- \frac{1}{ig}U((\partial_\nu U^{-1})(\partial_\mu U) - (\partial_\mu U^{-1})(\partial_\nu U))U^{-1} \end{aligned} \tag{53}$$

Since line 2,3,4 are cancelled by the last two lines 6 and 7 exactly what we want remains:

$$G'_{\mu\nu} = U(\partial_\nu B_\mu - \partial_\mu B_\nu)U^{-1} + igU[B_\nu, B_\mu]U^{-1} \tag{54}$$

This result was first produced by two American physicists Yang and Mills and that is why we call this Lagrangian the Yang-Mills Lagrangian. The Lagrangian derived from the $SU(2)$-symmetry was a Yang-Mills theory as well, however back then it was spontaneously broken. What we have produced is a Lagrangian with eight scalar fields in analogy with the $SU(3)$ symmetry. We will link these eight scalar fields to particles we call gluons. They are the carriers of the strong nuclear force. To get an idea about the representation of these gluons I have written down the matrix in which they can be presented, plus an example of the red-yellow mixing gluon:

$$\lambda_{general} = \begin{pmatrix} R\overline{R} & B\overline{R} & Y\overline{R} \\ R\overline{B} & B\overline{B} & Y\overline{B} \\ R\overline{Y} & B\overline{Y} & Y\overline{Y} \end{pmatrix} \tag{55}$$

$$\lambda_{red-yellow} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, g_{red-yellow} = \frac{-i}{\sqrt{2}}(r\overline{y} - \overline{r}y) \tag{56}$$

This shows that gluons carry colour as well. Furthermore, since there are no mass terms for the gluons we assume they are massless. They interact with the quarks as follows from Lagrangian (50):

$$L_{interaction} = \frac{-g}{2} b^n_\mu \overline{\psi_q} \gamma^\mu \lambda^n \psi_q \tag{57}$$

Then there are just a few more details to point out in this theory of the strong nuclear force. The first thing I would like to mention is the weird behaviour of the coupling strength, which we denoted at g. It has a very special dependence on distance, because when two quarks move away from each other the binding energy, carried by a gluon, increases. When a gluon is stretched it gains in energy like a rubber-band. This is not explained by anything in our theory, and remains one of the biggest questions in field theory physics today. A neat way of describing this phenomenon is by claiming quarks must always be in a colourless environment. What I mean to say that we can only find systems of $q\overline{q}$ or a set of three quarks all carrying different colours. Conventionally we call systems of two quarks mesons and systems of three quarks baryons. This is supported by experiments. It is impossible to extract a single quark from a system. When we try to, its energy increases because of the weird g-behavior and it creates another quark from the vacuum to form a new colourless system again.

The last thing I will have to introduce is quark mixing. When I introduced the weak scalar-fermion interaction in the previous section I claimed that they interacted by Yukawa interaction. This is true, however a slight modification must be made to the interacting quark doublets. They do not, as one expects, contain two quarks each but rather one quark and a linear combination of two other quarks:

$$\begin{pmatrix} u \\ d_\theta = d\cos\theta_W + s\sin\theta_W \end{pmatrix}_L \tag{58}$$

This mixing of quarks is proportional, again, to the weak mixing angle θ_W. In a theory based on just two quark families we can describe this mixing in matrix form like this:

$$U = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \tag{59}$$

This will transform strong-, or mass-eigenstates to weak-eigenstates and the other way round. This matrix has an analogue for more dimensions. The three dimensional case is of extra interest to us because there are currently three known quark families. There is strong evidence that there are just three families and that is why I will not give an extensive derivation of the N-case but just show it for $N = 3$. To transform a mass-eigenstate to a so called weak-eigenstate we introduce transformation matrices for each fermion. They leave the leptons and right handed quarks unchanged, because they do not have strong-eigenstates. The interesting matrices are then M^q_L. We use them to find translations between all possible quark interactions:

$$\overline{q'_i}\gamma_\mu q'_j = \overline{q_i}\gamma_\mu((M^{qi}_L)^\dagger M^{qj}_L)q_j = \overline{q_i}\gamma_\mu V_{ij} q_j \tag{60}$$

When performing this for all combinations of i and j we get a matrix known as the Cabbibo-Kobayashi-Maskawa mixing matrix, denoted V_{CKM}:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.04 \\ 0.01 & 0.04 & 0.99 \end{pmatrix} \tag{61}$$

The final matrix is filled with experimental values for the transition coefficients. We see that mixing between families is rare but certainly not zero. This is all there is to say about the local $SU(3)$ symmetry. This will bring us now to the next paragraph of this chapter where we will combine all results into L_{SM}!

3.2.7. The Standard Model Lagrangian

Now we will combine the results of the last two paragraphs and formulate one formula that forms the basis of HEP. In the formula below I have collected all terms we have derived since the beginning of this chapter. Going from top to bottom we see; the kinetic Higgs field term followed by the Higgs mass term, the fermion mass-terms and the fermion-Higgs coupling, the heavy boson mass terms, the charged and neutral weak interaction terms, the Higgs potential with condition $\mu^2 < 0$, the quadratic field tensor terms for the gluon and photon field and last but not least the quark-gluon interaction term plus the quark mixing term:

$$\begin{aligned}
L_{SM} = &\frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2\eta^2 - \sum_{f=0}^{n_f} \frac{G_f}{\sqrt{2}}(v+\eta)\overline{f}f \\
&+ \frac{g^2v^2}{8}(\mid W_\mu^+ \mid^2 + \mid W_\mu^- \mid^2) + \frac{1}{8}\frac{g^2v^2}{\cos^2\theta_W} \mid Z^0 \mid^2 \\
&+ \frac{-g}{\sqrt{2}}(\overline{f_L^0}\gamma^\mu f_L^1 W_\mu^+ + \overline{f_L^1}\gamma^\mu f_L^0 W_\mu^-) \\
&+ f^1\overline{f^1}\gamma^\mu f^1 A_\mu - \frac{g}{2\cos\theta_W}\overline{f_L^0}\gamma^\mu f_L^0 Z_\mu \\
&- \frac{g}{2\cos\theta_W}(2\sin^2\theta_W \overline{f_R^1}\gamma^\mu f_R^1 Z_\mu + (2\sin^2\theta_W - 1)\overline{f_L^1}\gamma^\mu \overline{f_L^1} Z_\mu) \\
&- \mu^2(\phi^\dagger\phi) - \mid \lambda \mid (\phi^\dagger\phi)^2 - \frac{1}{4}tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu}) \\
&- g_{QCD} b_\mu^n \overline{f}\gamma^\mu \lambda_\mu^n f + \sum_{q_L}^{n_q} \overline{q_L}\gamma_\mu V_{CKM} q_L
\end{aligned} \tag{62}$$

This is the result of building a theory starting with the Dirac equation, it is still Lorentz Invariant and $U(1)$-symmetric. Because of the reformulation of the $SU(2)_L \otimes U(1)_Y$-part we no longer recognize Y, but it is still a key part of the formula. Since we have included the confined QCD we can state that all physics that can be produced by this formula obeys the symmetries from the joined group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

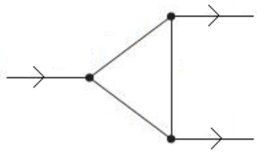
3.2.8. Partially broken symmetries

I will conclude this chapter with a few examples of symmetries from the Standard Model that are not in itself complete symmetries of the whole system. Studying symmetries like these will not result in new Lagrangian terms or physics we never knew. Rather it will help us to understand better what we do know and reveal hidden connections that are funded by experimental facts.
An example of this is the chiral symmetry. The Lagrangian seems to be invariant under independent transformation of left and right handed doublets, this is the group $SU(2)_L \otimes SU(2)_R$. But when we look closely we find that it is broken by the quark mass terms. One cannot independently transform left and right handed phases because of $L_{q-mass} = m_u \overline{u_L} u_R + m_d \overline{d_L} d_R + \cdots$. We can however transform left and right handed phases equally, $SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$, so these terms remain invariant. This is what we call the isospin symmetry: $SU(2)_C$. This is a straightforward analogue from (p, n) isospin, now related to the quarks from which p and n are built. But definitely not the same as weak isospin. As before, in the section on Higgs breaking, generators that spontaneously break the symmetry produce so-called Goldstone bosons. These bosons are massless particles that enter our theory. In the Higgs sector they were consumed in the mass terms of the heavy bosons. In this case, because the symmetry was not exact to begin with, these bosons do exist. They are known as very light particles called pions.

$$\pi^+ = \mid u\overline{d} >, \pi^0 = \frac{1}{\sqrt{2}}(\mid u\overline{u} - \mid d\overline{d} >), \pi^- = \mid d\overline{u} > \tag{63}$$

They consist of up and down quarks, as expected, because those were the ones that broke the symmetry.

Let′s look at another example of a approximate symmetry of the system. We might assume that the entire Lagrangian is symmetric not only under $e^{i\xi}$ but as well under $e^{i\gamma_5 \xi}$. These two symmetries we call vectorial and axial-vectorial. In group representation $U(1)_V \otimes U(1)_A$. When trying to create local symmetries from these global ones we find something interesting. The Noether current predicted by the vectorial symmetry is that of charge conservation. We do not know another experimentally seen conserved current that should be related to the axial-vector symmetry. However, in the case of first order interactions we find no problem in any of the possible processes and both J^μ_A and J^μ_V seem to be conserved. But when we look at more complicated interactions containing loops we find that the axial current is no longer conserved. For any process in which a particle decays into two other particles by means of a triangle diagram as shown below we find:



$$\partial_\mu J^\mu_A = \frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu} G_{\rho\sigma} [t_a, \{t_b, t_c\}] \tag{64}$$

.

The right side of the equation we call an anomaly, because it is not zero. The three generators in the commutator form the problem, each of them comes from one corner in the diagram. What we know from experiments is that these events do happen, which implies that $[t_a, \{t_b, t_c\}]$ should be zero for all possible generators of the Standard Model. When we compute $[t_a, \{t_b, t_c\}]$ for one of the possible diagrams, here we take the historically most profound one, we find the following: When a neutral pion decays into two photons by means of a triangle diagram the vertices can be of any fermion type. Nothing forbids this, pions actually decay to two photon with a strength predicted by such a diagram. Things are much worse when the anomalous symmetry is local. In such theories probabilities do not add up to one and we will have lost unitarity and renormalizability. It is therefore very important that the symmetries for the three forces we have considered are anomaly free! To test this, we should consider all possible allowed fermions in the loop, and generators on its vertices. When we consider there to be three $SU(2)$ generators we find easily:

$$[t_a, \{t_b, t_c\}] \propto \delta_{bc} Tr[t_a] = 0 \tag{65}$$

For $\{t_b, t_c\} = 2\delta_{bc}$ because the t_i are the Pauli matrices. In fact, when we compute all the possible diagrams for all possible fermion vertices we find they all cancel. In those calculation we have to use the fact that quarks appear in three colours which makes neutral pion decays an other argument for this to be true. So J^μ_A is not conserved but $e^{i\gamma_5\xi}$ is a global symmetry that manifests itself in certain interactions.

The last example of partially broken symmetries we will look at will lead us to another $SU(2)$ symmetry. When we look at the heavy boson mass terms we can introduce the new parameter ρ[5]:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + Corrections \tag{66}$$

This leads to a $SU(2)_{L+R}$ symmetry in the Higgs sector when we study diagrams leaving out loop diagrams. This is a isospin symmetry violated immediately by electromagnetism, but when taking out electromagnetism for a moment we learn something: We find that when we set θ_W to zero we can derive estimates for the different quark masses, they now are part of the leading correction terms. Since both estimates hold for very high energy scattering events this symmetry is used to predict the top quark mass. Measurements at Fermilab agreed with the predictions based on this hidden symmetry. Because we can learn quite a bit from this partially broken symmetry it has been given its own name; it is know as the custodial $SU(2)_C$ symmetry[7]. Custodial stands for custody, or safeguarding, of the corrections since they remain small.

This is where we end the section on the Standard Model, we have seen its derivation funded by local symmetries. We will continue to point out the other type of symmetries we know from nature and check what implications they have.

3.3. Global and Discreet

In this section we will focus mainly on symmetries we have not yet mentioned in the previous sections. We already saw some of natures global symmetries; Lorentz invariance, for example, is a global symmetry since it implies transformations equal for all points in space-time. The isospin invariance of nucleons and the definition of electrical charge both form global symmetries as well. Physical laws will remain the same and nature will not change under the operation of these symmetries. But we have already seen quite enough of these symmetries. They took part in the construction of the Standard Model where they were transformed into local symmetries. The custodial symmetry we saw on the previous page is a global symmetry as well.

In this section I want to focus on a different type of symmetry. In our first classes of classical mechanics we learn that quantities like energy and momentum are conserved. This is then linked by Noether's theorem to time invariance in classical physics. In quantum mechanics the argument is more subtle. Since energy is defined as the derivative of the wave function we can no longer say it is actually conserved but it is still linked to time invariance. Heisenberg's equation states that the more precise we try to measure the energy of a system the less precise our time measurement will be. We can never know the exact energy of a system at a given moment in time. This holds as well for the other well known Heisenberg pair of place and momentum. This is something we already expected because we knew that place invariance in classical mechanics implied momentum conservation. These are of course very pretty conserved quantities resulting from symmetries we have known for ages but when we think clearly they must have been violated. It is impossible to think of big bang theory and an expanding universe without violating energy and momentum conservation at some point. That is why we will take a closer look at global symmetries in respect to cosmology and especially the early universe.

These are a few facts we know about the universe: it is increasingly expanding in every direction and it has done so since the beginning of time itself. We assume that since this is the case the universe in a very early stage must have been very small and very hot. In physics today we try to understand what must have happened in the period between the big bang and the formation of the first stars. I say the formation of the first stars, because at that time the universe had come to a relatively quite and peaceful state. The period before that however must have been one of very fancy physics and complete chaos. We do not know much about it, but what we do know is we cannot know when it started. When it did, it violated every conservation law we know. Physicists today believe they can understand a pretty big part of the evolution of the universe and can tell us for example when the first particles were created and when these first formed baryons and mesons. And that brings us to the central question in this section of my paper: Why did those reactions back then bring forth more matter then anti-matter? All calculations that make predictions about that period of time are based on temperature and pressure estimates and statistical physics but nothing can tell us why the matter vs. anti-matter symmetry is broken.

This brings us back to global symmetries; what violates the matter vs. anti-matter symmetry and is it a fundamental global symmetry of nature? To address this we look at the difference of particles and anti-particles. We can describe an anti-particle by a normal particle moving back in time. Or just by a particle of opposite charge and with all other aspects like mass and helicity the same. The things we mention here are discrete global symmetries; Charge Parity and Time-reversal can all be switched. What interests us is whether nature would be different if we do switch those. In the next two paragraphs we will develop the theories that follow from these symmetries and combinations of them. Also we will have a look at what implications particle interactions have on them.

3.3.1. C, P and T

Like we said Charge, Parity and Time-reversal are all discrete symmetry-operations. Charge adds a minus sign to the charge of a particle, Parity to its direction and Time-reversal to its movement in time or equivalently to its momentum and spin. The combined operation of CPT is a complete symmetry of nature and there is no process we know of that violates this. One of its implications is that if our universe were completely CPT transferred it would evolve in the exact same way as ours. There would be no way to tell the difference.

Alone, however, all three symmetries are violated in weak decay processes of particle physics. Charge for example is violated by ordinary beta-decay since taking the charge conjugate of this process would imply the existence of right handed neutrinos. Since there are no right handed neutrinos in nature this is not a global symmetry at all. Even when neutrino oscillations are taking into account this argument holds. The same goes for Parity, there are no known examples of Parity reversed weak decay processes . They only decay in a way in which left handed neutrinos can be found. Although these symmetries hold for both electromagnetism and the strong nuclear force they do not for nature as a whole.

That just leaves us Time-reversal, would that be a symmetry of nature? For a long time everyone believed this was the case. It would, however, not explain the lack of anti-matter in the universe. Because when every reaction is the same going backwards, where did this difference come from? To answer this question we will look into the brother of Time-reversal symmetry; CP-symmetry. Since CPT is a global exact symmetry, when CP is violated so is T. They are actually the same but are historically known as CP.

3.3.2. What is CP?

CP is the symmetry operation which changes both Charge and Parity at the same time. What we will do in this section is first look at a few systems that violate CP and then check if we can find motivation for this violation in the Standard Model. Then, when we succeeded in both parts we will check what this means for cosmology.

CP is violated[8] . It is as simple as that, let us look at the following decay process: $K^0 \rightarrow \pi^0 + \pi^0$. The final state has CP eigenvalue +1. Let′s remember this and look into the eigenstates of K^0. There are two mass-eigenstates for K^0, both linear combinations of K^0 and $\overline{K^0}$: $K^1 = \frac{1}{2}(K^0 + \overline{K^0})$ and $K^2 = \frac{1}{2}(K^0 - \overline{K^0})$. Due to the minus sign K^2 has a opposite CP-eigenvalue with respect to K^1. When we look at decay processes of K^0 we see two different lifetimes, one significantly longer then the other. This supports our idea that K^1 and K^2 are different and actually exist in nature. The short living kaon always decays into pions. Since we know pions have CP state +1 so the short living kaon state should as well. This implies directly that the long living kaon should never decay into pions because that would mean a decay in which CP goes from +1 to −1. But this does happen! This is just a simple example of CP violation, we will get to know more decay processes in which this happens. I will just state now that the semi-leptonic neutral kaon systems also violate CP. The semi-leptonic decay of a neutral kaon goes like: $K_{short} \rightarrow \pi^- + l + \overline{v}$.

As we know from the section on the standard model, weak decays are a result of the following interaction term:

$$L_{weakdecay} = \frac{gW_\mu}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}\overline{q_j}\gamma^\mu q_i + \frac{gW_\mu^\dagger}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}^*\overline{q_i}\gamma^\mu q_j \tag{67}$$

Here we see that all physical information on the interactions must come from V_{ij} which is the CKM matrix. When we work on these terms with the CP operator we get the following:

$$\begin{aligned} &\frac{gW_\mu}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}\overline{q_j}\gamma^\mu q_i + \frac{gW_\mu^\dagger}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}^*\overline{q_i}\gamma^\mu q_j \\ \rightarrow &\frac{gW_\mu^\dagger}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}\overline{q_i}\gamma^\mu q_j + \frac{gW_\mu}{\sqrt{2}} \sum_{q_i,q_j} V_{ij}^*\overline{q_j}\gamma^\mu q_i \end{aligned} \tag{68}$$

These two lines would be the same if and only if $V_{ij} = V_{ij}^*$. This leads to a very simple conclusion; namely that CP violation can only be predicted by the Standard Model if the CKM matrix holds at least one fundamental imaginary component.

Group theory and some linear algebra give us a simple but powerful solution to this question. This was the key part of the paper for which half of the 2008 Nobel Prize was awarded, the other half was for work on spontaneously broken symmetries: Let′s assume we have a matrix V of dimension N by N. Maximally it then contains $2N^2$ parameters, assuming that each position can be taken by a complex element. Since the CKM matrix should be unitary all the columns of V should be orthogonal which implies a constraint for all $N(N-1)/2$ sets of columns with a real and imaginary part. Unitarity requires as well that there is a restriction for each column or row, because the absolute values must be real. Then looking at the quark side we have $2N$ quarks in N families, all but one carrying a relative phase. These do not have physical meaning and that leaves us with ...

$$2N^2 - N(N-1) - (2N-1) = N^2 - 2N + 1 = (N-1)^2 \tag{69}$$

$\ldots (N-1)^2$ real physical meaningful parameters. What we now want to know if V were real how many mixing angles like θ_W it would hold. It holds N^2 real elements, again the columns must be orthogonal which implies N(N-1)/2 constraints. Our theory give us an extra N constraints because we want the rows to be of unit length to keep everything renormalizable. Subtracting these constraints leaves us with $N(N-1)/2$ parameters for V. Now let's check the number of physically meaningful imaginary parameters of a matrix $V$with dimension N :

$$(N-1)^2 - \frac{N(N-1)}{2} = \frac{N^2}{2} - \frac{3N}{2} + 1 = \frac{1}{2}(N^2 - 3N + 2) = \frac{(N-1)(N-2)}{2} \qquad (70)$$

A remarkable result! We see that for two quark families we have no imaginary part. Furthermore it tells us that for three families, which we actually have seen, there must be a imaginary phase in the matrix which automatically produces CP violation as we have already shown!

3.3.3. Other SM CP-violations

Now we have seen CP violation for neutral kaon systems we wonder what else can violate CP. A natural system to look at is the third family counter part of the kaon, the neutral B-mesons. Neutral B-mesons are systems like $b\overline{d}$, $d\overline{b}$, $b\overline{s}$, $s\overline{b}$, and the extremely rare $b\overline{b}$. Since b-quarks are much more uncommon than s-quarks these reactions are much harder to study but that does not make measuring these decays completely impossible. At Fermilab and LEP proof has been found that these systems violate CP in a similar way as the neutral kaon systems do.

Then, looking back at our cosmological motivation for finding CP we wonder if these reactions can be responsible for the matter abundance in the universe today. This is unfortunately not the case, by just taking into account the weak CP violating decay we can only account for enough matter to create a single galaxy. Since this is not even one percent of the amount of CP-violation we need to fill the universe, there must be other reasons for the abundance of matter over antimatter.

Many physicists believed that there would be CP-violations in the QCD sector as well, but up to today there have not been any experimental results supporting this theory. It seems that the creation of the universe and the processes that took place back then will occupy our minds for at least some decades to come.

3.3.4. B and L

In my last theoretical words that can be accompanied by hard physical evidence I will discuss a global symmetry we have not jet studied. In reactions today we see the conservation of baryon- and lepton number. The lepton number was long thought to be conserved for each of the families separately but recent studies have proved the existence of neutrino oscillations. This means that neutrinos can oscillate between different families and thereby violate these symmetries. The B and L conservation follows straightforwardly from the $U(1)$-invariance of the Standard Model Lagrangian.

Interesting to note is that $B-L$ is conserved as well, but $B+L$ is not. This difference occurs in weak semi-leptonic decays where the minus sign in $B-L$ makes sure the conservation holds even under the creation of a pair of leptons.

Four decades ago Andrei Sakharov proposed the conditions that were necessary for the evolution of our galaxy in the first minute of its existence. This was based on the facts as they were known back then, CP violation had been found and the first measurements were made of the cosmic background radiation. These measurements indicated an asymmetry on cosmological scale and were supported by antimatter searching measurements. The cosmic background radiation was found not to be completely anisotropic and satellites that went looking for antimatter came back with zero results. One of Sakharovs conditions was CP violation. The others were reactions without thermal equilibrium due to the massive expansion of the universe itself and the violation of baryon number.

Baryon number can be violated by an anomaly process which occurs only in higher order calculations of $q\bar{q}$ annihilations.[9] This means that I should withdraw my statement of two paragraphs ago, the SM does not explain B and L conservation. Theory does not describe this conservation because of these anomalies. Experiments looking for these reactions, for instance in proton decays have not yielded any results. in the end, this means for us that the Standard Model is able to predict all interactions we see today in experiments but that it might need some alteration to describe events at the beginning of time.

4. DISCUSSION

4.1. Last gaps in the theory

The Standard Model, a theory of everything, a promising sound. Unfortunately, however, we have to tone this down. What the Standard Model Lagrangian is, is single formula that predicts all fundamental interactions of elementary particles in a quantum field vacuum. It does so under a few assumptions, beginning with the existence of the Higgs particle. As I said before, there has not yet been any direct physical proof of the Higgs field and because that is the very basis of the theory we are building on soft soil. But we are working on that. One of the four major experiments at CERN in Geneva is called ATLAS. ATLAS is specialized in finding the Higgs particle if it should exists. The confidence we have for this theory to be self-consistent and correct originates from every other experiment ever performed. That is why we now assume this theory to be correct and hope for ATLAS to confirm this. Other assumptions we make in the construction of the Standard Model seem less important. Taking the Dirac equation as a basis of this theory, introducing helicity and other axioms are necessary for our theory to be linked to nature. Experiments show us the values of the coupling constants we put into our theory. They are part of the natural way of doing physics. There are physicists who would like to see a simpler theory, consisting of less axioms and constants, but like Einstein said; " Everything should be made as simple as possible, but not simpler. "

That brings us to another part of physics, namely gravity, which has been completely left out of our theory. This has several reasons. In HEP gravity is of no importance. Gravity is a very, very weak force that we can only notice when we are in the neighbourhood of something very heavy like a planet or galaxy. Elementary particles do not even come close to the mass of a grain of sand, let alone a planet. But we would of course for esthetical reasons include gravity in the Standard Model if we could, because then it would be really a theory of everything. This is however fundamentally not possible in the way we describe the Standard Model. The theory of gravity, general relativity, describes particles it acts on not in a quantum field appropriate kind of way. This is of course very unfortunate and many theoretical physicists are working on combining these theories today. This shows again how difficult it is for us, not only practically, but also theoretically to learn about the creation of the universe since both elementary interactions and gravity play a central role in that field of physics.

4.2. Future prospects

It has always been fundamentally impossible to predict the future of physics. That is one of the reasons for building groundbreaking experiments like ATLAS at CERN. We certainly hope the Higgs boson is found but if we knew about its existence for sure we would never spend all that money. Exploring new magnitudes where we know nothing about is a dream for any explorer, as physicists are. We do not know what will happen. Everything we can do is building as many very exact detectors around the crossing points of particle beams and measure what happens when the highest energy collision ever created on earth takes place. Theoretical predictions can naturally be derived from the Standard Model and we assume that all measurements will agree with them. When the Higgs boson is found every particle physicist will be relieved and open a bottle of champagne. But if we still have not found any proof of its existence by the next ten years, we must really wonder where we went wrong. This is what makes it exciting times in particle physics.

Theoretical physicists are working all over the world to find new and extended theories that describe interactions on an even lower scale then the Standard Model does. But since there are no experimental facts about those unexplored, and maybe even unexplorable, regions there is no basis on which they can be judged. Those theories trying to extend the Standard Model in the other way, by including gravity, are known as the group of Grand Unified Theories. But likewise, since they have produced no results that can be seen as a valid extension of the Standard Model, the Standard Model remains the most powerful physical formula today, most certainly in High Energy Physics.

5. CONCLUSION

I started this project with an unfulfilled feeling about my theoretical particle physics classes, I wanted to know what actually governed the interactions of elementary particles. What I have shown is the method required to derive a single formula which does exactly this. That is why I am very pleased by the results this project has given me. We saw the construction of the Standard Model Lagrangian based on three symmetry groups from which we created covariant derivatives that produce boson fields. Then we introduced an extra, so far experimentally unproven, boson field who brought our theory in agreement with all experiments up to today. We have seen the different terms of the final Lagrangian and we were able to link them to the different particles and interactions that belong to quantum electrodynamics and the weak and strong nuclear force. The Standard Model Lagrangian, L_{SM} Eqs. (62), forms the very basis of HEP and was exactly the thing I was looking for. Together with the global symmetries we found for the Standard Model as a whole, this completed my understanding of the formulation and implications of particle physics.

6. A FINAL WORD

I have enjoyed working on this project for many weeks, after a slow start I got into the theory more and more and now after all those weeks it feels done. This project has helped me in seeing, even more so then before, the beauty that lies in physical formulations. This had never been possible without the guidance of my supervisor who introduced me to most of the physical concepts that I needed and always knew some more special symmetries that might be of interest to me. Thank you for all the help Eric! The other people I must thank are the ones who helped me in putting the gigantic pile of notes into this single .pdf file. Thanks to those who helped correcting my English and those who helped me figuring out the necessary LaTeX-tricks.

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